

# Foundations Of Quantum Theory Revisited \*

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## Abstract

After giving a panoramic view of the "text-book" interpretation of the new quantum mechanics, as a sequel to the old quantum theory, the conceptual basis of quantum theory since the Copenhagen Interpretation is reviewed in the context of various proposals since Einstein and Niels Bohr, designed to throw light on possible new facets bearing on its foundations, the key issues to its inherent "incompleteness" being A) measurement, and B) quantum non-locality. A related item on measurement, namely Quantum Zeno (as well as anti-Zeno) effect is also reviewed briefly. The inputs for the new facets are from some key Indian experts :

S.M. Roy, V. Singh ; D. Home; B. Misra; C.S. Unnikrishnan

## 1 Quantum Theory : Standard Picture

The issues concerning the foundations of quantum theory mainly hinge on two aspects, namely, A) quantum measurement problem, and B) quantum non-locality. To appreciate these subtleties, it is first necessary to have an overview of its "standard picture" [1] which is summarized below.

After the discovery by Max Planck of the existence of discrete quanta to account for the observed black-body spectrum, Quantum Theory got a big boost at the hands of Einstein himself who found a natural explanation of some crucial experimental observations on the photoelectric effect in terms of the particle nature of electromagnetic radiation, thus establishing its *dual* character. What was now needed was a viable atomic model to account for the discrete values of several measurable parameters of atomic systems, especially i) the Ritz classification of spectral lines in terms of the Rydberg - Ritz combination principle; and ii) Franck-Hertz experiment on the discrete energy losses of electrons on collision with atoms. To that end, the right background was provided by Rutherford's discovery of the atomic structure, and a crucial step was taken by Niels Bohr who postulated that i) atomic systems can only exist in certain quantized states, each corresponding to a well-defined energy, so that transitions between them are accompanied by radiation whose energy ( $E$ ) equals their energy difference ; and ii) the frequency of the radiation quantum is equal to  $E/\hbar$ . These two postulates sufficed for an understanding of both the Rydberg-Ritz combination principle, and the Franck-Hertz experiment. Bohr's postulates received a further boost through a quantization rule discovered by Wilson and

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Sommerfeld within the Hamilton-Jacobi formulation of classical mechanics, namely, the (classically derived) action integrals must be integral multiples of  $\hbar$ , so that the corresponding energy levels, expressed in terms of the action integrals, should automatically have quantized values. This so-called " Old Quantum Theory " was highly successful in explaining a huge mass of spectroscopic data , such as the fine structure of the hydrogen atom, the spectra of diatomic molecules, without further assumptions. But there were many difficulties in the way of a 'natural' understanding of the Old Quantum Theory.

## 1.1 Difficulties with Old Quantum Theory

The difficulties [1] were both i) practical and ii) conceptual. i) Practical, because this theory was not applicable to aperiodic systems, not did it properly account for the *intensities* of spectral lines, and with improvement in experimental techniques, the gap between experiment and theory increased. [ The Correspondence Principle was introduced by Bohr to ensure better agreement in the limit of large quantum numbers when the classical conditions are more valid, but it was at most a stop gap arrangement]. ii) Conceptual, because a conceptually satisfactory explanation of the basic phenomena was lacking. [ For example it was difficult to understand why the Coulomb force in the hydrogen atom was so effective for the spectroscopy, while the ability of an accelerated electron to emit radiation disappeared in a stationary state]. And the assumption of a dual character of light (particle-like on emission and absorption, and wave-like in transit) seemed to lack logical self-consistency. Most of these problems disappeared with a more elaborate approach under the name of New Quantum Mechanics.

## 1.2 New Quantum Mechanics

The advent of the New Quantum Mechanics was preceded by several conceptual breakthroughs in quick succession. First, de Broglie (1924) showed through his postulate  $\lambda = h/p$  that wave - particle duality was not a monopoly of radiation alone, as emanating from the Planck-Einstein discoveries. This concept was equally applicable to material particles, as was soon to be demonstrated by the Davisson-Germer (1927) and G.P. Thompson (1928) experiments. Secondly S.N. Bose (2) introduced the concept of *indistinguishability* through a new mode of counting for the derivation of Planck's law, a result which, as noted above, was endorsed by Einstein through a corresponding derivation for material particles [3]. The concept of indistinguishability which stemmed from the Bose form of counting, had no "classical" counterpart, and led to the prediction by Einstein of Bose-Einstein condensate, one in which a finite fraction of particles trickle down to the lowest state [3]. Bose's concept also found a natural echo in the Matrix Mechanics of Heisenberg who abandoned the concept of individual particle identity that had characterized classical mechanics, in favour of *operators* (as matrices) for dynamical variables like position ( $q$ ) and momentum ( $p$ ) on the one hand, and *states* (as vectors) on which the dynamical variables operate to produce their "measured" values (in a representation labelled by suitably discretized variables) on the other. Thus the individual values of dynamical variables in classical mechanics now gave way to an *array* of numbers (the matrix elements) in their matrix representation. In the new approach, the problem of finding the *values* now reduced to one of 'diagonalization' of these matrices, the diagonal elements being interpreted as the only results of *measurement*, viz., *eigenvalues*, and

the corresponding vectors being interpreted as the possible *states*, viz., *eigenstates*, on which the measurements were valid. This concept of *measurement* in turn necessarily led to the (dual) concept of object- observer pair, thus vastly extending the domain of the physics beyond the corresponding classical domain which had no scope for an observer as a separate dynamical entity.

### 1.3 Uncertainty Principle and Complementarity

The outcome of this new formalism [1,4-5] may be summarized by the so-called *Uncertainty Principle (UP)* of Heisenberg (1927), according to which it is impossible to specify precisely and simultaneously the values of both members of *canonically conjugate* pairs of dynamical variables (like  $(q, p)$ ) that describe the behaviour of atomic systems. Some other pairs of canonically conjugate variables, in an obvious notation, are  $(J_z, \phi)$  and  $(E, t)$ . The uncertainty relations for typical pairs in an obvious notation are :

$$[\Delta x \times \Delta p_x; \Delta \phi \times \Delta J_z; \Delta t \times \Delta E] \geq \hbar/2$$

which imply that the determination of any dynamical variable with unlimited precision must be traded with a corresponding loss of information of its canonically conjugate momentum, and vice versa. And the smallness of  $\hbar$  makes the result of physical interest only for systems of atomic size or less. In particular, an energy determination with accuracy  $\Delta E$  must cost a time interval of at least  $\sim \hbar/2\Delta E$ , which implies, e.g., that energy conservation in a certain process can be violated with impunity provided the time available for measurement is short enough ! As a word of caution, however, the time-energy UP should be distinguished from the other UP's, since a formal time operator is not defined in conventional quantum mechanics, as first shown by Pauli.

For a physical understanding of the implications of *UP*, Bohr introduced the *Complementarity Principle* which states that atomic phenomena cannot be described with the completeness demanded by classical mechanics. Now certain pairs (the canonically conjugate ones !) that complement each other for a complete classical description, are actually mutually exclusive, although they are needed for a full description of various aspects of the phenomenon. On the other hand, from the experimenter's point of view, the complementarity principle asserts that the physical apparatus available to him is so constrained that more precise measurements than those mandated by the *UP* cannot be made. This must not be regarded as a limitation of the experimenter's techniques, but a more intrinsic law of nature which dictates that whenever an attempt is made to measure one of a pair of canonical variables, the other is changed by an amount that cannot be estimated without interfering with the primary attempt. This is fundamentally different from the classical situation, in which the measurement process in principle disturbs the system under observation no doubt, but the amount of disturbance is either too small to be of consequence, or it can be calculated and taken into account; in either case the 'disturbance' due to measurement is *not* of much *dynamical* significance. Not so for an atomic system, whose behaviour cannot be described independently of the means by which it is observed, so that the object and the observer are inextricably linked together in a dynamical fashion.

Stated differently, the situation for an atomic system is as follows : One must choose between various experimental arrangements, each designed to measure the two members of a pair of canonical variables with different degrees of precision that are compatible

with the *UP*. In particular, there are two extreme arrangements, each of which measures one member of the pair with great precision. In the ‘classical’ theory, these extreme experimental arrangements complement each other; the results of both are available simultaneously, and indeed are necessary for a complete description of the system. But in a ‘quantum’ theory which actually applies to the atomic system, such extreme complementary experiments are mutually exclusive, and cannot be performed simultaneously. Thus the classical concept of causality is no longer valid in a quantum situation: There is causality to the extent the quantum equations of motion are perfectly well defined. But the causal relationship between successive configurations of an atomic system that characterize a classical description, no longer exists. And the role of *measurement* is now an active part of the quantum dynamics.

## 1.4 Simple Diffraction Experiment [1, 5]

The way New Quantum Mechanics ” resolved ” the inner contradictions of the Old Quantum Theory is best illustrated by the interpretation of a simple diffraction experiment which though text-book material [1], has acquired a renewed significance [4, 5] in the context of recent developments on the very foundations of modern quantum theory.

Consider a double-slit experiment of the standard type [1, 5] (figure omitted for brevity, but the notation of [5a] is kept in the following). A light source *S* is placed in front of a screen *A* in which two parallel slits  $A_1, A_2$  are cut in horizontal directions; a second screen *M* parallel to (and behind ) *A*, is equipped with proper devices to measure the pattern. Now consider three different cases: 1) particles; 2) waves; 3) electrons as follows [5].

### 1.4.1 Particles

When the source shoots (classical) point particles, say bullets, measure the vertical distribution  $P$  of the fraction of the number of bullets arriving at *M*. In this case each bullet goes through either slit  $A_1$ , or through  $A_2$ , and arrive at a definite point on the screen *M*. Let the vertical distribution be  $P_1$ , if  $A_1$  is open;  $P_2$ , if  $A_2$  is open; and  $P_{12}$ , if both are open. In this case,  $P_{12} = P_1 + P_2$ , a commonsense result which in the modern language goes by the name of a ” decoherence ” effect [5b] due to multiple interactions of the ‘bullet’ with the environment, resulting in a clear ‘which slit ’ passage for its motion (see below).

### 1.4.2 Waves

Next consider water waves produced at the source *S*, and measure the vertical intensity  $I$  of the wave motion which takes on the values  $I_1, I_2, I_{12}$  for the three cases as above, respectively. This time a full-fledged (albeit classical) wave-like scenario ensures a typical interference pattern ( via the principle of superposition) for the resultant intensity distribution when both slits are open, so that  $I_{12} \neq I_1 + I_2$ .

### 1.4.3 Photons

Now consider the source *S* emitting radiation , to illustrate the quantum scenario. Here we have the radiation behaving like a wave during its passage from the source *S* to the screen *B* via the slit *A*, but behaving like quanta (photons) when ejecting photoelectrons

from B, so that the wave and particle aspects appear in the same experiment. What is the distribution now ? We find a diffraction pattern, namely  $I_{12} \neq I_1 + I_2$  ! Is it due to the interference between different photons passing through the two slits ? This explanation is not sufficient, since the diffraction pattern still appears when the intensity of light is so much reduced that on average, only one photon at a time passes from the source to the screen, which must be through one of the two slits only. So one must conclude that the diffraction pattern is a statistical property of a *single* photon, and *not* due to interference of more than one photon. This leads one to ask: " how does the presence of a slit through which the photon does not go, prevent it from reaching a part of the screen that it would be likely to reach if that slit were closed ?" [1]

Quantum mechanics " resolves " the issue [1] through the assertion that the diffraction pattern would be destroyed if a sufficiently careful attempt were made to determine through *which* slit each photon passes. Thus if we place a detector C near one of the slits to find out if it passed through that slit, we find indeed that  $P_{12} = P_1 + P_2$ , i.e., the diffraction pattern is destroyed [1] ! In the modern language of Decoherence which may be defined as a process by which the environment destroys the wavelike nature of things by getting information about a quantum system [5b], the interaction with the detector C introduces a 'decoherence' effect resulting in a clear 'which-slit' passage for the electron. But in the absence of this decoherence, the diffraction pattern is restored as if the photon were a wave only. A more conventional explanation is that any attempt to find out through which slit the photon has passed gets into direct conflict with the Uncertainty Principle [1].

## 2 Anatomy Of Quantum Theory : New Facets

The foregoing is a text-book background [1] on the ramifications of Standard Quantum Mechanics vis-a-vis the classical theory, illustrated with a familiar diffraction experiment. Before going into its depths, it is perhaps in order to stress that the first victim of the quantum paradigm was the Cartesian Partition between the *physique* and the *psyche*, since a *measurement* (involving as it did, a close interaction between the two) now became a key ingredient of the new physics, which severely restricted the hitherto "ontological" (out there) status of a classical observable. Initially therefore it was quite hard for the Western physics community to adjust to this sudden change in paradigm. This was best illustrated by Einstein's profound unhappiness [6] with the 'incompleteness' of quantum mechanics as revealed, for example, by the diffraction experiment. Curiously enough, it did not seem to sound such an unfamiliar ring with "eastern thought" , with its penchant for mysticism (!), as was once to be revealed to L. Rosenfeld during his discussions with Japanese physicists on the subject. Einstein's famous conversations with the poet Rabindranath Tagore also pointed to a similar divergence of emphasis between the ontological and philosophical aspects of reality ( for details see Home [4]).

The other aspect of the foundations of quantum theory is that while such studies were for long regarded by many to be of "metaphysical" importance only [4], have suddenly sprung up over the last 2-3 decades as frontline items of study, thanks to the growth of new experimental techniques on the one hand, and to the development of *quantum technologies of information processing and transferring* on the other . This has led to spectacular advances in quantum optics technology and quantum information processing.

Therefore it is of great physical interest to investigate the manner in which the abstract conceptual issues of quantum mechanics can be linked to the actual experiments in order to obtain new insights, as well as to uncover some of the unexplored facets of quantum mechanics.

## 2.1 Classical Physics Ontology [5]

To bring a semblance of order to the turbulence caused by the sudden paradigm shift, it is useful to have a critical reassessment of the classical premises prior to the quantum formulation. Two basic hypotheses which the Greek thinkers made about Nature, according to Schroedinger (quoted in Singh [5a]) are: 1) the existence of a real external world, which amounts to taking the observer's consciousness out of the purview of the observed world; 2) this external world is accessible through the existence of laws of nature (read Newton's laws of classical mechanics).

Next one must emphasize the *unitary* nature of classical mechanics, namely [5a], it describes both the system under observation, the measuring apparatus as well as their mutual interaction. The interaction disturbs the system in principle, but the disturbance can be reduced to any desired level of accuracy, so that the measurement is not an epistemological problem, rather a practical one. Thus classical physics (unlike quantum mechanics) does *not* require a split between the system to be observed, and the observing apparatus. And the basic ontological "entities" of classical physics are *point – particles* and *fields* (waves) , both moving in 4D space-time.

### 2.1.1 Determinism

The laws of classical physics are *deterministic* and causal, namely the equations of motion with specified forces predict both the past and future, subject only to the initial conditions to any desired order of accuracy. It is another matter that the classical behaviour of a large number of classical particles is for practical purposes more conveniently described by "statistical" mechanics, but there is no epistemological aspect to such a strategy. A further set-back to determinism arises from *chaotic* classical dynamics which avers that arbitrarily close phase points can diverge away exponentially from each other under dynamical evolution, so that even carefully determined initial conditions do not necessarily guarantee the future behavior to any desired accuracy. An apparent loss of determinism also comes from Brownian (random) motion in a fluid due to collisions by a large number of molecules in a fluid ! An interesting observation ( Singh [5a]) is that Einstein's expression for the root mean square displacement of a Brownian particle has the same algebraic structure as Heisenberg's *UP*, except that it involves the diffusion coefficient of the fluid, instead of  $\hbar$ , in it.

The laws of classical physics are centred around a commonsense concept of "locality" whose significance is best manifested by our capacity to deal meaningfully with the external world in a piecemeal fashion, and not all at once. Indeed, it is enough to identify independent subsystems of the external world to any desired order of accuracy, and deal only with them, while ignoring the rest, since any two subsystems which are too far apart, do not affect each other appreciably. And locality and determinism in turn are crucial to our concept of "classical realism" [4] whose basic tenets are as follows :

C1) All physical attributes of an individual object have definite values associated with

them at any instant of time *irrespective* of their actual measurement (which is necessarily non-invasive).

C2) Realism in the classical macro-world is intimately linked to *Causality*, in the sense that the values of physical attributes of an individual object at different instants of time are uniquely connected by the relevant laws.

## 2.2 Quantum Reality: EPR Theorem [6]

In quantum mechanics, on the other hand, if two systems have once interacted together, and later separated, no matter how far, they can no longer be assigned separate state vectors. A famous example is a spin-zero object at rest, breaking up spontaneously into two fragments,  $A_1, A_2$  with spins  $S_1, S_2$  respectively, moving in opposite directions. By conservation of angular momentum, the two spins must be equal and opposite, so that any measurement of one (say  $S_1$ ) will automatically fix the value of  $S_2$ , even without any explicit measurement ! This situation goes much against intuition, since a physical interaction between these two objects, receding far away from each other, is negligible. According to Schroedinger [5], this is a most important characteristic of quantum mechanics. ( In the modern jargon, this effect is known as " Entanglement " [5b] ; see further below ). And in a famous paper entitled " Can quantum mechanical description of Reality be considered complete (?)", EPR [6, 5] considered this paradoxical aspect of quantum mechanics, by sharpening it via two definitions:

i) A necessary condition for the completeness of a theory is that " every element of the physical reality must have a counterpart in the physical theory".  
 ii) A sufficient condition to identify an element of physical reality is " if without in any way disturbing the system, we can predict with certainty the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity".  
 The result of these considerations was the EPR Theorem [6] , namely, the *incompatibility* of the following two statements:

- 1) the description in terms of the  $\psi$ -function of quantum mechanics is complete;
- 2) the real states of spatially separated objects are independent of each other

The second statement is called the "Einstein locality postulate". But then the first statement which says that quantum mechanics is a complete description, is clearly incompatible with Einstein locality. This incompatibility is called the EPR theorem. And, the correlations in Quantum mechanics, which are necessarily *non – local* inasmuch as they do *not* decrease with distance, are due to *quantum entanglement* which violates Einstein's locality.

Thus there is a conflict between classical and quantum realisms, which calls for a more precise formulation pending an experimental test. To begin with, corresponding to the classical realism tenets C1 and C2 [4] given above, analogous tenets Q1 and Q2 of quantum realism may be defined as follows [4]:

Q1) Reality cannot be associated with the *unobserved dynamical attributes* of microphysical entities;

Q2) It is not in general possible to determine the "state" of a Quantum system, or the values of its dynamical attributes without affecting the system's subsequent time evolution. Thus a measurement on a quantum system is necessarily *invasive*. Later, in Sect. 5, we give a fuller discussion on this subject in the context of the celebrated Bell's Theorem [7].

### 3 Quantum Entanglement & Contextuality

The invasiveness of a quantum measurement, in turn, necessitates some formal concepts which arise in the description of "quantum ontology" [4, 5(a)] in terms of some "classical" models of quantum reality, proposed since the "Copenhagen Interpretation". Two principal concepts associated with the new paradigm are those of i) quantum entanglement [5(a, b), 8-9], and ii) "quantum contextuality" [10], as prerequisites for a description of this subject.

#### 3.1 Quantum Entanglement [8 - 10]

This is an aspect of quantum theory, which is already implied in the EPR theorem [6], but has come to the fore because of its simultaneous linkage to measurement problems as well as to quantum non-locality. Indeed the discussion in connection with the EPR theorem [6] above already shows that if two systems interact, then no matter how far they get separated, their states get inevitably *entangled* (non-factorizable) [9], leading to a non-trivial quantum measurement problem on the following lines [10].

Consider a system initially in a state ( $a\psi_1 + b\psi_2$ ) as a superposition of two states  $\psi_1, \psi_2$  which are the eigenstates of a dynamical variable to be measured. The interaction of this system with a measuring device, results in a final state of the form

$$\Psi = a\psi_1\Phi_1 + b\psi_2\Phi_2$$

where  $\Phi_1$  and  $\Phi_2$  are two mutually orthogonal and macroscopically distinguishable states of the device. This final state  $\Psi$  is necessarily *entangled* [8-10], which implies that no separate state can be ascribed to the apparatus (whose function is to register the outcome of a measurement by associating it with the property of the apparatus). Now  $\Psi$  is a pure state, i.e., it corresponds to an ensemble of identical members, each of whose members is described by the *same* wave function  $\Psi$ , with a *single* outcome of measurement. On the other hand, all measurements result in a final ensemble of systems coupled to apparatuses corresponding to *different* outcomes of measurement, implying thereby that a post-measurement ensemble is necessarily *heterogeneous*, i.e.,  $\Psi$  is a *mixed* state.

Here is a genuine problem: How to generate a "mixed" state from a "pure" state within the formalism of standard quantum mechanics, since under no unitary time-evolution can a pure state evolve into a mixed state! Since on the other hand, a mixed state is a necessary condition for the occurrence of definite outcomes of measurement, which are *distinguishable* from other outcomes, it must be different from a "pure" state of the form  $\Psi$  above. The problem of its 'realization' is thus the essence of the Quantum measurement problem whose acuteness was highlighted by Schroedinger through his famous "Cat Paradox" [8]. In the next Section we attempt to outline several approaches for circumventing this problem, (starting from Bohr!), and arrive at some workable methods for quantum ontology.

#### 3.2 Quantum Contextuality & Non-Contextuality

"Quantum contextuality" which characterizes a quantum measurement, arises as follows [11]. A quantum state vector  $|\psi\rangle$  specifies the probability  $|\langle\alpha|\psi\rangle|^2$  of observing the set of eigenvalues  $\{\alpha\}$  of a complete set of observables  $A$  in the experimental situation or



"context" where  $A$  is observed. Equally,  $|\psi\rangle$  specifies the probabilities  $|\langle\beta|\psi\rangle|^2$  for observing the eigenvalues  $\{\beta\}$  of a different set  $B$  in the "context" where  $B$  is measured. Each context corresponds to the experimental arrangement to measure one complete set of observables. Due to this *inherent* context dependence, quantum mechanics does *not* specify joint probabilities of non-commuting observables. It is usually assumed that  $A$  and  $B$  cannot be simultaneously measured if they contain mutually non-commuting observables, which means that  $|\langle\alpha|\psi\rangle|^2$  and  $|\langle\beta|\psi\rangle|^2$  refer to *different* "contexts". Thus quantum probabilities are inherently context - dependent. Further, the context dependence is *irreducible*, i.e., quantum mechanics cannot be imbedded in a classical "context - independent" stochastic theory. . This is the essence of the Gleason-Kochen-Specker theorem [12-13]. These contextuality theorems circumscribe the extent to which dynamical variables in quantum mechanics can be ascribed simultaneous "Reality", independent of observations.

Similarly the hypothesis of "non-contextuality" is the assumption that the outcome of a measurement of a dynamical variable, say  $A_1$ , is taken to be the same irrespective of any other observable (commuting with  $A_1$ ) measured with it [14]. Such variables have been given the name "beable" in Bell's terminology [15], whose values are independent of the context of observation, whereas other variables may have context-dependent values.

### 3.3 Quantum Locality vs. Non-Locality

Unlike classical locality (Sect 3.1.2), which does not extend beyond a common-sense definition, quantum locality is much harder to define since it is inextricably linked with non-locality effects. Roughly speaking, quantum non-locality denotes quantum mechanical action at a distance (termed 'spooky' by Einstein) in a situation where a distant influence appears to be counter-intuitive because of the absence of any classically describable form of physical mediation [10].

More precisely, a non-local effect means affecting the state of an individual entity by any form of dynamical intervention in a faraway region such that no known physical influence can causally connect the occurrences in *that* region to the system under study [10]. For a space-like separation, a quantum non-locality necessarily implies entanglement for spatially separated particles.

## 4 Quantum Ontology

The various "interpretations" of the quantum formalism fall broadly into two classes, I) those that are supplemented by extra ingredients (necessitated by the compulsions of preparing a "mixed state" [10] !); and II) those that are not. Class I includes the Copenhagen group of interpretations (Bohr, Heisenberg, von-Neumann et al), as well as the de-Broglie - Bohm causal interpretations. Class II includes Everett's relative - state and many-world approaches, as well as the Quantum History approach initiated by Griffiths, Omnes, Gellmann & Hartle, and Zurek [5].

### 4.1 Class I Ontology : Bohr

Bohr took an intensely pragmatic view, (based on logical priority), by giving a workable definition for the *measurement* process. He insisted that the language of classical physics

is the only one available for communicating the observed results of any phenomenon, so that all measuring devices must be described by classical physics as a *matter of principle*. This amounts to supplementing the formalism of quantum mechanics with the addition of classical physics to describe the measuring apparatus. Thus [5a] the combined system  $S$  and measuring apparatus  $A$  is an unanalyzable whole phenomenon ( $S + A$ ). For example ( $S + A_1$ ) representing the electron  $S$  and its position recorder  $A_1$ , is different from ( $S + A_2$ ) standing for the electron  $S$  and its momentum recorder  $A_2$ , so that both cannot be combined into a single description corresponding to an electron with a known position and momentum. This is how the uncertainty principle is incorporated in practice: the two descriptions ( $S + A_1$ ) and ( $S + A_2$ ) are complementary. The quantum world is probabilistic, and a measurement does not reveal any pre-existing property of the system. Bohr was against drawing any further ontological picture of the quantum world, beyond the measurement process. And due to the overwhelming success of the New Quantum Mechanics, this *Copenhagen Interpretation* of Bohr had many more followers (who were more anxious to work out the multi-dimensional consequences of the new theory ) than those sharing Einstein's misgivings, so that the conceptual issues emanating from the sense of incompleteness of quantum mechanics never got off the ground until rather recently, when the prospects of experimentation have become brighter.

## 4.2 Class I Ontology : Heisenberg & Von Neumann

Bohr did not assign any particular significance to the wave function  $\psi$  of the quantum system in his "No Ontology" view. In contrast, in Heisenberg's formulation, the wave function  $\psi$  plus all observables  $O(t)$ , represents "objective tendencies" for actual events to occur. Thus while the wave function  $\psi(t)$  represents the wave-like aspects of nature, the particle-like aspects are represented by "actual events". This implies the use of the concept of the "collapse of the wave function" : The actualization of the tendencies happens when the experimental result is recorded, e.g., a click in the geiger counter. In the words of Heisenberg [5a], "The observation itself changes the probability function discontinuously; it selects, of all possible events, the actual one that has taken place", i.e., the wave function has "collapsed", a concept that Von Neumann articulated more explicitly.

In von Neumann's view, the measuring apparatus  $A$  as well as the system  $S$  are both to be described by quantum mechanics. Thus one has now two kinds of eigenfunctions and eigenvalues, one for the system  $S$  [ $\psi_n, \omega_n$ ], and one for the apparatus  $A$  [ $f(a_n), a_n$ ]. Von Neumann now postulates the "measurement interaction" which causes the initial system-apparatus state  $\psi_n f(a)$  to evolve into  $\psi_n f(a_n)$ . Then by the superposition principle of quantum mechanics, if the system is initially in a superposition  $\sum_n c_n \psi_n$ , the measurement interaction will cause the evolution :

$$\sum_n c_n \psi_n f(a) \Rightarrow \sum_n c_n \psi_n f(a_n)$$

As a result, the probability of observing the pointer reading to be  $a_n$ , corresponding to the system being in the state  $\psi_n$ , will be  $|c_n|^2$ .

So far, it is only quantum mechanics, giving the state  $\sum_n c_n \psi_n f(a_n)$  without a specific value of the pointer reading. Von Neumann now postulates that when the measurement is completed, the wave function now *collapses* to a *single* term, namely,  $\psi_N f(a_N)$ . It is this collapse postulate that acts as the extra ingredient to quantum mechanics.

### 4.3 de Broglie-Bohm Causal Interpretation

While we are still on Class I types of Quantum Ontology, we digress into a special type of the latter for a more detailed presentation since it has been the subject of very extensive investigations during recent times. It started with Louis de Broglie who, in 1927, proposed a realistic causal interpretation of quantum mechanics [16] in his "pilot wave" theory, but it did not find ready acceptance in the smug "Bohr-filled" atmosphere prevailing at the time. Then in 1952, David Bohm [ 17 ] came up independently with a similar proposal, overcoming the earlier objections to the de Broglie proposal, and found support from no less a person than John Bell, presumably because it tied up with his concept of "beables" [15]. [ The requisite "beable" in this case is the *position* variable  $x$  with a special status ]. After Bell's support, there has necessarily been intense activity in this field, with some major Indian contributions [18-21]. We summarise some essential features of this theory, together with recent developments

The ontology of de Broglie - Bohm theory is "realistic" [5a]. This means that a quantum object, say an electron, has both a particle aspect ( a trajectory  $q(t)$  associated with it), and a wave aspect viz., its wave function  $\psi(q, t)$  is involved in determining its velocity  $\dot{q}(t)$  through a relation of the form

$$m\dot{q}(t) = \nabla S$$

where  $S$  is the phase of the wave function  $\psi$  and the time evolution of the latter is described by the Schroedinger equation. Thus the usual quantum formulation is *supplemented* by the addition of particle trajectories, so as to make it "deterministic". Which means that, given position  $q(t)$  and wave function  $\psi(q, t)$ , at  $t = 0$ , it can be predicted at all later times. Then what is the significance of quantum probabilities in this scenario ? They arise through our inability to precisely control particle positions. Indeed, the best we can do is to prepare a statistical ensemble at  $t = 0$  in which the particle position is distributed according to the probability distribution

$$P(q, t = 0) = |\psi(q, t = 0)|^2$$

and subsequently the probability distribution evolves in accordance with the de Broglie-Bohm dynamics, namely

$$P(q, t) = |\psi(q, t)|^2$$

How does this scenario gel with the standard wave-Particle duality ? The answer is that it is *not* a particle or wave according to the experimental set-up ( a la standard quantum mechanics ), but that it is *both* a particle and a wave. In the example of the double slit experiment, the "particle" of course goes through one of the slits, but the "wave" goes through both.

It should *not* be concluded from this that we have reverted to classical determinism, since the electron trajectories in the de Broglie-Bohm theory are quite different from those in standard classical physics. Thus, e.g., in the double slit experiment, the trajectories never cross the middle plane between the slits ! To put it crudely though, this theory appears as a sort of 'insertion' (by hand) of an element of classical mechanics into the quantum picture.

#### 4.3.1 Recent Developments : Roy-Singh Approach

The above is a broad outline of the de Broglie-Bohm "causal" theory, which also has many non-classical features, as the above discussion shows. In recent years it has been generalized in several ways, but before discussing such generalizations, it is pertinent to recall Wigner's famous  $(q, p)$  distribution [22] based on standard quantum mechanics. Unlike de Broglie-Bohm theory, the Wigner distribution gives a *symmetrical* treatment to the position and momentum variables, albeit at the cost of positive-definiteness, as has been shown by several people [23]. Roy and Singh [18] set about formulating a generalization of the de Broglie-Bohm theory designed to give a symmetrical treatment of position and momentum, while bravely preserving the positivity condition on the joint  $(q, p)$  distribution function, ( through suitable  $\delta$ -functions), wherein the apparent violation of the above theorem [23] is sought to be reconciled via some further insertions (by hand) on the deBroglie-Bohm picture. Roy-Singh have termed their formulation as a "maximally realistic" description of causal quantum mechanics.

#### 4.4 Class II Ontology : Everett's " Many Worlds"

With no extra assumptions beyond quantum mechanics, Everett proposed an interpretation in which the Schroedinger equation provides a complete description of Nature [24]. Since the measuring apparatus is described by quantum mechanics, there is no " collapse " of the wave function in this approach. It is a unitary description without any split between the system and the apparatus. B. S. de Witt [25] popularised this interpretation as the " many worlds interpretation " of quantum mechanics. The idea is broadly as follows [5a] :

Because of the time evolution implied in the Schroedinger equation, an observer would be expected to be in a superposition of wave functions describing different eigenstates. If, e.g., we are in a normalized state

$$\Psi = c_1\psi(\textit{walking east}) + c_2\psi(\textit{walking west})$$

, as a result of the interaction, we should be simultaneously walking to the east as well as to the west. Everett asserts that this is precisely what actually happens : Due to the measurement interaction, we split into *two* editions of ourselves; one walking to the east, and the other walking to the west ! Thus the universe has split into *two*, with probabilities  $|c_1|^2$  and  $|c_2|^2$  respectively. But we are not aware of this split, as we seem to live in only *one* universe.

The theory maintains objective reality, but only at the cost of a vast proliferation of the universes, since the universe splits every time an observation is made. On the other hand, the wave function never collapses as an observation is made; only its different components inhabit different universes. And despite extravagant (and wasteful) use of universes, it has an economy of principles, since the only input is the Schroedinger equation.

This interpretation has found an appeal to those working in Quantum Computation, since [26] the massive parallel computing associated with quantum computation fits in with the language of the superposition

$$\Psi = \sum c_j\psi_i$$

if we regard that a computation on each  $\psi_i$  is carried on in a different universe.

## 4.5 Quantum History Approach [27-30]

This approach, which is associated with the work of Griffith [27], Omnes [28], Gell-Mann - Hartle [29], and Zurek [30], may be regarded as a sort of minimal (logical) completion of the Copenhagen approach, to make up an organic whole. Its essential ingredients may be summarized a la [5a] as follows :

- 1) A possible set of fine-grained histories  $h$ ;
- 2) A notion of coarse graining;
- 3) A decoherence functional  $D(h, h')$  defined for each pair of histories  $h$  and  $h'$ , satisfying standard mathematical properties.
- 4) A Superposition principle for coarse graining defined as a standard sum over histories  $h, h'$ .
- 5) A decoherence condition for two histories  $h, h'$ .
- 6) A natural definition of the probability  $p(h)$  for the history  $h$  as  $p(h) = D(h, h)$ .

How does the ‘quantum histories approach’ solve the measurement problem ? It circumvents the problem of wave function collapse by asserting that ” the measurement problem stems from the presence of those parts of the wave function corresponding to those alternatives that do not actually happen ” [5b]. And when can we ignore the alternative parts of the wave function ? The answer is equally simple : The other parts of the wave function can be ignored ” at exactly the moment when they have no further effect on us ” [5b]. From this point on, the future history of the wave function *decoheres* [5b] . Thus the interpretation provides for quantum probabilities only for histories in *decoherent* sets.

## 4.6 Anything Else ? Afshar Experiment & After

The galaxy of ” interpretations” described above for quantum theory, all swear by the *mathematics* of the Shroedinger Equation or its relativistic (Dirac) generalization, on which there is no controversy from any quarter. The ‘problem’ seems to arise when the different *interpretations*, from the Copenhagen to the Many Worlds interpretation, seem to co-exist ! Now while the Bell formulation (see next Section) marked a big step towards distinguishing between the ‘classical’ vs ‘quantum’ predictions – and found an *experimental* resolution of the issue, thanks to the Aspect experiment –, so far no corresponding experimental way to distinguish among the different ‘intellectual’ candidates for physical interpretation of the *same* mathematics of the quantum theory, seems not to be forthcoming. And without such a concrete mechanism, there seems to be no way to prevent further proliferation of more such alternatives. Against this background, the latest Afshar experiment [PRL 2004] has opened up a fresh opportunity for a possible resolution of the issue.

The main thrust of the Afshar experiment<sup>1</sup>, is to posit a concrete test of the Copenhagen Interpretation on the impossibility of observing both particle and wave properties in the same experiment. This he does through an ingenious variant of the famous two-slit experiment wherein he inserts one or more thin wires at the previously measured positions of the interference minima to define the *which – way* routes for the passage of the photons through a pair of pinholes, in an otherwise identical set-up to the canonical arrangement. In one such setup, if the wire plane is uniformly illuminated, the wires absorb about 6% of

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<sup>1</sup>as narrated in J.G. Cramer, <http://www.npl.washington.edu/TI>

the light. Then Afshar measures the difference in the light received at the pinhole images with and without the wires in place. What should he observe ? Both the Copenhagen and Many Worlds interpretations expect the wires to intercept about 6% of the light they do for uniform illumination. But actually Afshar observes almost *zero* % interception, for, wires placed at the zero-intensity locations of the interference minima intercept no light, thus implying that the wave interference pattern is still present despite an unambiguous "which-way" definition of the passage of light photons through a pair of pinholes, a feat hitherto considered to be impossible[ Neils Bohr, Nature **121**, 580 (1928)]. But from this one need *not* jump to the conclusion that the mathematical theory of quantum mechanics has also been falsified. For, a simple quantum mechanical calculation using the standard formalism shows that the wires should intercept only a very small fraction of the light. So while the formal mathematics of quantum mechanics is fully vindicated, it is these 'interpretations' of quantum mechanics that may need refinement, unless specifically tailored to meet the objections [ J. Cramers, Reviews of Modern Physics **58**, 647 (1986)] !

## 5 Bell's Theorem [6]; [10]

After this short detour on quantum ontology, we come back to a systematic study by John Bell of the consequences of the EPR theorem [6]. The result of this study which goes by the name of Bell's theorem [7], is the most famous legacy of John Bell, as it draws an important line between quantum mechanics (QM) and the world as we know it intuitively. Despite its simplicity and elegance, it touches upon many of the fundamental philosophical issues that relate to modern physics. In its simplest form, Bell's theorem, which has been called "the most profound in science" (Stapp, 1975), states:

No physical theory of local hidden variables can ever reproduce all of the predictions of quantum mechanics.

To appreciate the significance of this terse Theorem it is first necessary to analyse the ingredients which Bell harnessed to arrive at this result. For the background, Bell examined both the EPR paper [6] and John von Neumann's 1932 proof of the incompatibility of hidden variables with QM. As to the former, since Einstein's locality postulate appeals instantly to intuition, Bell's analysis [7] was designed to make the concept of Einstein locality more precise by introducing "hidden variables" as a means of circumventing the counter-intuitiveness implied in quantum entanglement ; on the other hand, he was also conscious of Von Neumann's "no go theorem" on quantum mechanics vis-a-vis hidden variables. Taking a generalized view of 'hidden variables', he set about formulating the ramifications of the EPR theorem in the form of certain mathematical inequalities (called "Bell's inequalities") as a concrete expression of the requirement that the assumption of local realism - that particle attributes have definite values independent of the act of observation, and that distant objects do not exchange information faster than the speed of light - leads to, in respect of certain types of phenomena which do not exist in quantum mechanics. And one way to define 'hidden variables' is to say that they are synonymous with the well defined properties associated with these inequalities.

The "inequalities" concern measurements made by remotely located observers (often called Alice and Bob ) on entangled pairs of particles that have interacted and then

separated. Assuming hidden variables, they lead to strict limits on the possible values of the correlation of subsequent measurements that can be obtained from the particle pairs. Bell discovered that these limits are outside the predictions of quantum mechanics in special cases. Quantum mechanics(QM) does not assume the existence of hidden variables associated with individual particles, and so the inequalities do not apply to it. The QM predicted correlation is due to quantum entanglement of the pair, with the idea that their state is not determined until the point at which a measurement is made on one or the other. This idea is fully in accordance with the Heisenberg uncertainty principle , a basic tenet of QM.

If one accepts Bell's theorem: either quantum mechanics is wrong, or local realism is wrong, since they are mutually incompatible. To address the issue scientifically, experiments needed to be performed over the years, with concomitant improvements in technology. Bell's test experiments to date overwhelmingly show that the inequalities of Bell's theorem are violated [31]. This is taken by most physicists as providing empirical evidence against local realism, while constituting positive evidence in favor of QM; but the principle of special relativity is saved by the no-communication theorem , which proves that it is impossible for Alice to communicate information to Bob (or vice versa) faster than the speed of light.

## 5.1 Bell's Inequalities

The above is as far as the qualitative logic of Bell's Theorem goes. For a more quantitative description, the following sketch from Home [10] is instructive. Consider two spin- 1/2 particles in a singlet state, and moving in opposite directions towards two measuring Stern-Gerlach magnets. The corresponding wave function is an "entangled" (non-factorable) state [10]:

$$|\psi\rangle = [|\alpha\rangle_1 |\beta\rangle_2 - |\beta\rangle_1 |\alpha\rangle_2] / \sqrt{2} \quad (1)$$

where  $\alpha$  and  $\beta$  are spin-up and spin-down states respectively. Measurements of the components of spins  $\mathbf{s}_1, \mathbf{s}_2$  along two different directions are performed on these particles. For particle 1, let  $\hat{a}$  and  $\hat{a}'$  be the unit directions along which its spin is measured; then  $A = 2\hat{a} \cdot \mathbf{s}_1$ , and  $A' = 2\hat{a}' \cdot \mathbf{s}_1$  are its spin components in these directions, whose measured values are  $\pm 1$ . Similarly for the particle 2, the quantities  $B = 2\hat{b} \cdot \mathbf{s}_2$ , and  $B' = 2\hat{b}' \cdot \mathbf{s}_2$  are the corresponding spin components whose measured values are again  $\pm 1$ .

Now consider the linear combination  $AB + A'B + AB' - A'B'$ . For any given pair of spatially separated particles 1 and 2 in the singlet state, eq.(1), one can measure only *one* of the products  $AB, A'B, AB', A'B'$ . In each case the answer must be  $\pm 1$ . The experiment consists in making measurements on a large number of such pairs, with the setting on one wing (particles 1) alternating between  $\hat{a}$  and  $\hat{a}'$ ; and on the other wing (particles 2) between  $\hat{b}$  and  $\hat{b}'$ . In this way an ensemble of measurements of each of the quantities  $AB, A'B, AB', A'B'$  is performed, and the final experimental data are their "average values"  $\langle AB \rangle, \langle A'B \rangle, \langle AB' \rangle, \langle A'B' \rangle$ .

To evaluate the average values, Bell made two assumptions : **A** an individual outcome of a particular measurement is definite, and not affected by what outcome is obtained by measurement in a region which is sufficiently separated from the entity under study. [This is the "locality" condition for individual events] ; **B** the randomly chosen sample of pairs on which a quantity like  $AB$  is measured, is typical of the entire ensemble. These two assumptions together lead to a testable constraint on the correlation functions, *without*

any input from quantum mechanics. Indeed, assumption **A** implies that one can associate definite measured values with both the spin components  $A(\pm 1)$  and  $A'(\pm 1)$  for each particle 1; these values are independent of whether or not  $B$  or  $B'$  is measured on particle 2. Similarly for  $B, B'$  vis-a-vis  $A, A'$ . Therefore each particle pair has a value  $\pm 1$  of each of the measured quantities  $AB, A'B, AB', A'B'$ . Then, for each of the 16 different cases corresponding to the possible choices  $\pm 1$  for each of  $A, A', B, B'$  separately, the resultant of the pair combinations gives

$$AB + A'B + AB' - A'B' = \pm 2 \quad (2)$$

where it is understood that both the occurrences of, say,  $A$ , in the lhs of (2), have the *same* value; and similarly for  $A', B, B'$ . [This is the locality condition once again]. Summing (2) over the entire ensemble of pairs, and taking the average, one gets

$$| \langle AB \rangle + \langle A'B \rangle + \langle AB' \rangle - \langle A'B' \rangle | \leq 2 \quad (3)$$

where, by virtue of **B**, the various averages can be identified with the experimentally measured values of the corresponding correlated quantities. This is Bell's inequality which is a verifiable prediction involving measurable quantities, as a direct consequence of the locality condition.

## 5.2 Quantum Violation of Locality

This inequality is violated by the quantum mechanical results for the singlet state which yields  $\langle AB \rangle = -\hat{a} \cdot \hat{b} = -\cos\theta$ . The maximum violation occurs when all directions are coplanar, with  $\hat{a}' \cdot \hat{b}' = -1/\sqrt{2}$ , and the other 3 cosines are all  $1/\sqrt{2}$ . For these values the rhs equals  $2\sqrt{2}$ . There are of course a whole range of angles over which the above inequality is violated quantum mechanically.

What is the significance of this result ? It is that, irrespective of the specifics of any causal model, no causal theory satisfying the locality condition, can be fully consistent with the formalism of quantum mechanics – a classic "no-go theorem" ! Indeed a decisive experimental refutation [31] of Bell's inequality has a non-trivial implication of *irreducible quantum inseparability for macroscopic separations*. This is a repudiation of the cherished notion of the principle that any composite extended system may be regarded as composed of elements which are localized in separate regions, in favour of an indivisible whole !

## 6 Responses to Quantum Violations

Possible responses to quantum violations of Bell's theorem are: a) give up the assumption that  $\psi$  provides a complete description of the state of an individual entity; b) a viable causal model of quantum mechanics must be *non-local*; c) Other forms of locality. Possibility (a) points to the need for modifying the formalism, say with "hidden" variables to further specify the state of an individual microsystem, so as to make it compatible with the quantum measurement problem. Possibility (b) raises the question of compatibility with Lorentz invariance.



## 6.1 Hidden Variable Effects

As response (a) to the quantum violations of Bell's theorem, we may incorporate hidden variable effects, within the above formalism [10]. Denoting hidden variables by  $\lambda$ , the amplitude  $A$ , e.g., may be represented by  $A(\hat{a}, \lambda)$ , to show its dependence on  $\lambda$ ; similarly for the other amplitudes  $A', B, B'$ . Then the result of measurement  $A(\hat{a}, \lambda)$  of the  $\hat{a}$  spin-component of the first particle  $2\mathbf{s}_1.\hat{a}$  would be  $\pm 1$ , i.e.  $A(\hat{a}, \lambda) = \pm 1$ . Similarly, the result of measurement  $B(\hat{b}, \lambda)$  of the  $\hat{b}$  component of spin of the second particle  $2\mathbf{s}_2.\hat{b}$  would be given by  $B(\hat{b}, \lambda) = \pm 1$ . Now Einstein locality implies that  $A$  does not depend on  $\hat{b}$ , nor does  $B$  depend on  $\hat{a}$ . One further stipulates that the measurement epochs are such that no direct light signals can travel between the two Stern-Gerlach magnets.

The spin correlations are now given by the average value  $\langle \hat{a}, \hat{b} \rangle$  of the product  $AB$  :

$$\langle \hat{a}, \hat{b} \rangle = \int d\lambda \rho(\lambda) A(\hat{a}, \lambda) B(\hat{b}, \lambda) \quad (4)$$

where  $\rho(\lambda)$  is the non-negative normalized probability distribution of the hidden variables for the given quantum mechanical singlet state  $\psi$ ; and [5a]

$$|A(\hat{a}, \lambda)| \leq 1; |B(\hat{b}, \lambda)| \leq 1$$

From these representations one shows the Bell's inequalities [5a]

$$| \langle \hat{a}, \hat{b} \rangle - \langle \hat{a}, \hat{b}' \rangle | + | \langle \hat{a}', \hat{b}' \rangle + \langle \hat{a}', \hat{b} \rangle | \leq 2$$

On the other hand, quantum mechanics predicts

$$\langle \hat{a}, \hat{b} \rangle = \langle 2\mathbf{s}_1.\hat{a} 2\mathbf{s}_2.\hat{b} \rangle = -\hat{a}.\hat{b}$$

This last form of  $\langle \hat{a}, \hat{b} \rangle$ , whose modulus usually exceeds 2, cannot therefore satisfy the Bell's inequality for a arbitrary choice of directions. Then it follows that no local hidden variable theory can reproduce all the results of quantum mechanics. This is once again Bell's Theorem.

Bell's inequalities are tested by "coincidence counts" from a Bell test experiment. Pairs of particles are emitted as a result of a quantum process, analysed with respect to some key property such as polarisation direction, then detected. The settings (orientations) of the analysers are selected by the experimenter. The source S produces pairs of "photons", sent in opposite directions. Each photon encounters a two-channel polariser whose orientation (a or b) can be set by the experimenter. Emerging signals from each channel are detected and coincidences of four types (++, --, +- and -+) counted by the coincidence monitor. Bell test experiments to date overwhelmingly suggest that Bell's inequality is violated [31] The phenomenon of quantum entanglement that is implied by violation of Bell's inequality is just one element of quantum physics which cannot be represented by any classical picture of physics; other non-classical elements are complementarity.

### 6.1.1 Single Particle Correlations

But experiments with two correlated photons need not be the only possible way to show quantum mechanical violations of orthodox paradigms. An interesting experiment was proposed by Home et al [32] with single photon states incident on two prisms facing

each other, designed to show simultaneous particle and wave characteristics in apparent violation of Bohr's complementary principle in its usual form. The performed experiment [33] verified their quantum optical prediction [32] .

Coming now to Bell's inequalities, experiments beyond two-photon correlations [31] may also include material particles. A recent *single* neutron interferometry experiment [34] showed violation of Bell's inequalities by measuring correlations between *two* degrees of freedom (comprising spatial and spin components) of *single* neutrons, (thus obviating the need for a source of entangled neutron pairs).

## 6.2 "Unnikrishnan Locality"

A radical departure from these "orthodox" responses to quantum violations of Einstein locality is provided by a different line of thought proposed by Unnikrishnan [35] that local amplitudes with random initial phases can be assigned to individual particles of a correlated entangled system and that the quantum correlations are encoded at source in the relative phase of these amplitudes. This has the dramatic effect that Einstein locality is not violated during observations on entangled system, contrary to the widespread standard belief ! This "Unnikrishnan locality" ( see B. d' Espagnet [36]) claims to preserve Einstein locality through its insistence on *conservation laws to predetermine* the correlations.

## 7 Quantum Zeno Effect And All That

We now come to a topic which is closely related to the measurement syndrome, but whose philosophical implications can be traced all the way to the Greek philosopher Zeno, as it goes by the name of the quantum Zeno effect [37-40]. It is a seemingly paradoxical result in quantum theory concerning the slowing down of the evolution of a dynamical system under repeated observation of an unstable system over a period of time, so that its decay can be greatly inhibited. Although the idea in the quantum context dates back to Schroedinger [37], the present interest in the subject owes its origin to the seminal work of Misra and Sudarshan [40]. More recently, it was predicted that repeated measurements can even *enhance* the decay [41 - 43], a phenomenon which was termed *Anti-Zeno* effect. The experimental observation of these effects relies on the ability to reset the evolution of the system during the non-exponential time of the decay . We first outline the Misra-Sudarshan [40] theory for non-exponential decays giving rise to the Zeno effect, after which we briefly indicate the possibility of anti-Zeno effects, together with experimental results [44].

### 7.1 Unstable Quantum System : Definitions [40]

There are 3 main ingredients for the quantum description of an unstable system : i) a Hilbert space  $\mathcal{H}$  of state vectors, including the unstable states and their decay products; ii) a unitary group  $U_t = \exp -iHt$  acting on  $\mathcal{H}$  to describe its time evolution; iii) the subspace  $\mathcal{M}$  of  $\mathcal{H}$  formed by the undecayed unstable states of the system. The orthogonal projection onto  $\mathcal{M}$  is denoted by  $\mathcal{E}$ , a two-valued observable that corresponds to the "yes-no" experiments to determine if the system is in an undecayed or decayed state respectively. In terms of these quantities, the probability  $P(t)$  for finding the system

undecayed at time  $t$ , if it was prepared in the state  $\rho$  at  $t = 0$ , is given by  $P(t) = \text{Tr}[\rho U_t^* \mathcal{E} U_t]$ , while the probability that at the instant  $t$  the system is found decayed, is the complementary quantity  $Q(t) = \text{Tr}[\rho U_t^* \mathcal{E}^\perp U_t]$ , such that  $P(t) + Q(t) = 1$ , and  $\mathcal{E} + \mathcal{E}^\perp = 1$ .

While the quantities  $P(t)$  and  $Q(t)$  which correspond to specific instants of time, are unambiguously given by the standard rules of quantum mechanics, they are by themselves inadequate for constructing  $P(\Delta, \rho)$  and  $Q(\Delta, \rho)$  which represent the probabilities that the system prepared initially in the undecayed state  $\rho$  will be found to decay or remain undecayed (respectively), *sometime during or throughout (respectively) a given interval*  $\Delta = (0, t)$  [45]. And till the Misra-Sudarshan paper [40], quantum theory did not have any ready formula for  $P(\Delta, \rho)$  or  $Q(\Delta, \rho)$ , on the basis of which they were led to conjecture that quantum theory might well be incomplete ! They proceeded to investigate the problem as follows [40].

## 7.2 Repeated and Continuous Observations

They looked for the operational meaning of such probabilities over *extended* time intervals in terms of the outcomes of continuous monitoring of the unstable particle for its existence in the undecayed states. Now continuous monitoring may be regarded as a limit of frequently repeated observations as the dead time between successive observations tends to zero. They further argued that that there is no bar in quantum theory *per se* against such idealization since if were so, it would imply "discreteness of time" ! They then attempted to construct the probability  $P(\Delta, \rho)$  as the limit of the corresponding probability under successive measurements, as the interval between them approaches zero.

As a beginning, consider only three measurements at times  $t = 0, t/2, t$ , and seek the probability that the system initially prepared in a given state  $\rho$  will be found in a decayed state in at least one of these three measurements. According to the normal rules, this probability is the *sum* of the following three: 1) the probability for a decayed state at  $t = 0$ ; 2) the conditional probability for a decayed state at  $t = t/2$ , given that the system was undecayed at  $t = 0$ ; 3) the conditional probability for decay at  $t = t$ , given that the two preceding measurements had left the system undecayed. Now the first probability is just  $P(0) = \text{Tr}[\rho \mathcal{E}^\perp]$ . But the other two (conditional) probabilities require the knowledge of the *state changes* caused by measurements at  $t = 0, t/2$ , involving *collapse* of the state vector (or reduction of wave packet), caused by the very process of measurement [ see sect. 4.2 ], which needs a short digression.

## 7.3 Collapse of State Vector

The initial state  $\rho$  collapses due to measurement at  $t = 0$ , to the state  $\rho' = \mathcal{E} \rho \mathcal{E}$ . Thereafter, its evolution until the second measurement at  $t = t/2$  is governed by  $U_{t/2}$ . Thus the system, after being found to be undecayed at  $t = 0$  is in the state  $\rho'' = U_{t/2} \rho' U_{t/2}^*$ . Hence the conditional probability for finding the system in a decayed state at  $t = t/2$  when it was undecayed at  $t = 0$  is [40]

$$P(t/2; 0) = \text{Tr}[\mathcal{E}^\perp \rho'' \mathcal{E}^\perp] \quad (5)$$

In the same way, the conditional probability  $P(t; t/2)$  works out as

$$P(t; t/2) = \text{Tr}[\mathcal{E}^\perp U_{t/2}^2 \rho (U_{t/2}^*)^2 \mathcal{E}^\perp] \quad (6)$$

The total probability for finding the system decayed is the sum of these 3 conditional probabilities which simplifies after algebraic rearrangements, to

$$P(\Delta, \rho)_2 = 1 - \text{Tr}[\rho(\mathcal{E}U_{t/2}\mathcal{E})^{*2}(\mathcal{E}U_{t/2}\mathcal{E})^2] \quad (7)$$

The law is now fairly clear. Therefore, generalizing the sequence, one can compute the probability for finding the system in a decayed state in at least one of the sequence of  $(n + 1)$  measurements undertaken at  $t = (0, t/n, 2t/n, \dots, t)$  in the form

$$P(\Delta; \rho)_n = 1 - \text{Tr}[\rho(\mathcal{E}U_{t/n}\mathcal{E})^{*n}(\mathcal{E}U_{t/n}\mathcal{E})^n] \quad (8)$$

whose limit at  $n \rightarrow \infty$  is expected to be the desired probability  $P(\Delta; \rho)$ .

## 7.4 Quantum Zeno Paradox

To obtain the  $n \rightarrow \infty$  limit of the above, it is necessary to study the quantity

$$\lim_{n \rightarrow \infty} (\mathcal{E}U_{t/n}\mathcal{E})^n \equiv T(t)$$

Then it was shown by Misra-Sudarshan [42] that under some general conditions like semi-boundedness of the Hamiltonian,

$$T^*(t)T(t) = \mathcal{E} \quad (9)$$

Substitution of this result in (9) shows that in  $n = \infty$  limit,  $P(\Delta; \rho) = 1 - \text{Tr}(\rho\mathcal{E})$ , so that it is *independent* of  $t$  ! So if the initial state is undecayed, i.e.,  $\text{Tr}(\rho\mathcal{E}) = 1$ , then  $P(\Delta; \rho) = 0$  for all intervals  $\Delta$ . Thus one arrives at the paradoxical conclusion that the probability for an unstable particle will be found in a decayed state at *some* time during  $(0, t)$ , is *zero*, no matter how large  $t$  is. This is the quantum Zeno paradox [42], which is more picturesquely expressed by the statement that an unstable quantum system under continuous observation, *does not decay* !

The startling nature of this result naturally raises doubts on its validity. Therefore unless a more reliable way than the above [40] is found for the derivation of  $P(\Delta; \rho)$ , the very issue of completeness of quantum mechanics [1] would remain open to question. Note also that the above derivation has made essential use of the concept of "collapse of the wave function" [type I ontology, Sect. 4] as part of the measurement process.

## 7.5 Deviations from Exponential Decay

Deviations from exponential decay in quantum mechanics were predicted by Khalfin [45] who showed that if the Hamiltonian has a spectrum bounded from below, then the survival probability  $P$  is *not* a pure exponential, but rather of the form  $\exp(-ct^q)$  where  $q < 1$  and  $c > 0$ . More generally, Winter [46] showed that the survival probability begins with a non-exponential oscillatory behaviour, after which the system evolves according to the exponential law, and finally tapers off like an inverse power of time. A similar result was found by Chiu et al [47] who showed that the time parameters  $T_1, T_2$  separating the above three domains bear the inequalities  $T_1 \ll \Gamma^{-1} \ll T_2$ , where  $\Gamma^{-1}$  is the total life-time for decay. And the quantum Zeno of [40] refers to the time period  $T_1$  only. The initial non-exponential decay behaviour is related to the fact that the coupling between the decaying

system and the reservoir is reversible for short enough times, for which the decayed and undecayed are not yet resolvable [44]. Indeed the Misra-Sudarshan theorem [40] can also be understood from the fact that, given a finite value of the mean energy of the decaying state, the survival probability  $P(t)$  obeys the relation [48]

$$\{dP(t)/dt\}_{t=0} = 0 \quad (10)$$

which is a general property independent of the details of the interaction.

## 7.6 Quantum Zeno vs Anti-Zeno

The Misra-Sudarshan prediction [40] of the quantum Zeno effect was restudied [41-43] by focussing on the frequency of observations, and on the decay of an unstable system as a consequence of the existence of a reservoir of possible states. The result was the prediction of the *opposite* effect. Namely, repeated observations must shorten the life-time of an unstable system – the anti-Zeno effect ! Both types were observed by Medina et al [44] in an unstable system in the forms of inhibition and enhancement respectively, by frequent measurement during the non-exponential time.

## 8 Quantum Optics & Information [49]

This narrative will remain incomplete without some comments on the Quantum Information Theory that has developed dramatically over the past two decades, driven by the prospects of quantum-enhanced communication and computation systems. It exploits the unique properties of quantum mechanics which facilitate different ways of communication, processing and information. For one thing, a quantum system must evolve coherently, which means that it must remain isolated from the external environment. But to process the (quantum) information, it must have strong interaction with classical measuring systems and control elements. An important property of a quantum information is that it cannot be cloned, since any attempt to ‘touch’ it will destroy it, via the Uncertainty Principle. Thus quantum entanglement is an essential ingredient of any communication task. On the positive side, a computer algorithm can be more efficiently processed than a classical system. Indeed, soon after Feynman’s prediction (1987) of this feature, Shor gave an algorithm for the determination of prime factors which can be processed much faster by a quantum computer than with a classical one. A related algorithm due to Grover [50] facilitates a faster searching ( $\sqrt{N}$ ) of a data-base of  $N$  items than one ( $N/2$ ) by classical means.

The unit of quantum information, like the classical, is based on a binary system of quantum states (like  $|0\rangle, |1\rangle$ ), and is measured in *qubits* (information digitally encoded on a quantum system), in analogy to classical *bits*. But there exist more possibilities in a quantum system, such as the bases can be  $|\pm\rangle$  which are orthogonal combinations of  $|0\rangle, |1\rangle$ . Further, qubits can span different bit values simultaneously. This is trivially true of a single qubit, but also holds for multi-qubit states. However, in a quantum system, different bases do *not* commute. Thus a measurement in one basis disturbs the bit values in another basis. This feature can be employed for a secure communication channel via *quantum key distribution* or QKD. [This is also called quantum cryptography].

## 8.1 Quantum Teleportation

Let us illustrate the new ideas in quantum communication through the working of Quantum Teleportation, or entanglement-assisted teleportation. It is a technique that transfers a quantum state to an arbitrarily distant location using a distributed entangled state and the transmission of some classical information. The key theoretical ingredients involved are EPR driven quantum entanglement, and (suitable manipulation of) Bell states (see Sect 7). Quantum teleportation does not transport energy or matter, nor does it allow communication of information at superluminal speed.

In the standard nomenclature in quantum information: the two parties are Alice (A) and Bob (B), and a qubit is in general a unit vector in two-dimensional Hilbert space. Suppose Alice has a qubit in some arbitrary quantum state

$$|\psi_o\rangle = \alpha|0\rangle + \beta|1\rangle \quad (11)$$

. Assume that this quantum state is not known to Alice, and she would like to send this state to Bob. Simple options like i) physically transporting the qubit to Bob, or ii) broadcasting the information, are not viable, since fragileness of quantum states rules out (i) ; and a "no-cloning theorem" forbids (ii). Again *classical* transportation (result of her measurement communicated to Bob, who then prepares the qubit in his possession accordingly) is impossible since quantum information cannot be measured reliably. Thus, Alice has a seemingly impossible problem on hand. A solution was discovered by Bennet et al. [50]: The parts of a maximally entangled two-qubit state are distributed to Alice and Bob. The protocol then involves Alice and Bob interacting locally with the qubit(s) in their possession and Alice sending two classical bits to Bob. In the end, the qubit in Bob's possession will be in the desired state. The logic goes as follows.

Suppose Alice has a qubit that she wants to teleport to Bob. Our quantum teleportation scheme requires Alice and Bob to share a maximally entangled (EPR driven) state beforehand, for instance two 2-particle Bell states

$$|\Phi^\pm\rangle = [|0\rangle_A \otimes |0\rangle_B \pm |1\rangle_A \otimes |1\rangle_B] / \sqrt{2} \quad (12)$$

Alice takes one of the particles in the pair, and Bob keeps other one. The subscripts A and B in the entangled state refer Alice's or Bob's particle. So, Alice has two particles (O, the one she wants to teleport, and A, one of the entangled pair), and Bob has one particle, B. In the total system, the state of these three particles is given by:  $|\psi_o\rangle \otimes |\Phi^+\rangle$ , with the above definitions. Alice will then make a partial measurement in the Bell basis on the two qubits in her possession. To make the result of her measurement clear, we will rewrite the two qubits of Alice in the Bell basis via the following general identities expressed in pairs in a compact notation :

$$\begin{aligned} |0\rangle \otimes |0\rangle; |1\rangle \otimes |1\rangle &= [\Phi^+ \pm \Phi^-] / \sqrt{2}; \\ |0\rangle \otimes |1\rangle; |1\rangle \otimes |0\rangle &= [\Psi^+ \pm \Psi^-] / \sqrt{2} \end{aligned} \quad (13)$$

The three-particle state  $|\psi_o\rangle \otimes |\Phi^+\rangle$ , when expanded, then becomes:

$$\begin{aligned} |\Phi^+\rangle_A \otimes [\alpha|0\rangle + \beta|1\rangle]_B &+ |\Phi^-\rangle_A \otimes [\alpha|0\rangle - \beta|1\rangle]_B \\ |\Psi^-\rangle_A \otimes [\alpha|1\rangle + \beta|0\rangle]_B &+ |\Psi^+\rangle_A \otimes [\alpha|1\rangle - \beta|0\rangle]_B \end{aligned}$$

Notice all we have done so far is a change of basis on Alice's part of the system. No operation has been performed and the three particles are still in the same state. The actual teleportation starts when Alice measures her two qubits in the Bell basis. Given the above expression, evidently the results of her (local) measurement on the total system is that the three particle state would collapse to one of the following four states (with equal probability of obtaining each):

$$|\Phi^\pm\rangle_A \otimes [\alpha|0\rangle \pm \beta|1\rangle]_B; \quad |\Psi^\pm\rangle_A \otimes [\alpha|1\rangle \pm \beta|0\rangle]_B$$

Alice's two particles are now entangled to each other, in one of the four Bell states. The entanglement originally shared between Alice's and Bob's is now broken. Bob's particle takes on one of the four superposition states shown above. Note how Bob's qubit is now in a state that resembles the state to be teleported (the four possible states for Bob's qubit are unitary images of the state to be teleported). The crucial step, the local measurement done by Alice on the Bell basis, is done. It is clear how to proceed further. Alice now has complete knowledge of the state of the three particles; the result of her Bell measurement tells her which of the four states the system is in. She simply has to send her results to Bob through a classical channel. Two classical bits can communicate which of the four results she obtained. After Bob receives the message from Alice, he will know which of the four states his particle is in. Using this information, he performs a unitary operation on his particle to transform it to the state  $[\alpha|0\rangle + \beta|1\rangle]$ . If Alice indicates her result is  $|\Phi^+\rangle$ , Bob knows his qubit is already in the desired state and does nothing. This amounts to the trivial unitary operation, the identity operator. If the message indicates  $|\Phi^-\rangle$ , Bob would send his qubit through the unitary gate given by the Pauli matrix  $\sigma_3$  to recover the state. If Alice's message correspond to  $|\Psi^+\rangle$ , Bob applies the "gate" (unitary transformation, that is) via Pauli matrix  $\sigma_1$  to his qubit. Finally, for the remaining case, the appropriate "gate" is given by  $\sigma_3\sigma_1 = i\sigma_2$ . Teleportation is therefore achieved. Experimentally, the projective measurement done by Alice may be achieved via a series of laser pulses directed at the two particles.

After this operation, Bob's qubit will take on the state  $[\alpha|0\rangle + \beta|1\rangle]$ , and Alice's qubit becomes an (undefined) part of an entangled state. Thus, teleportation does not result in the copying of qubits, and hence is consistent with the no-cloning theorem. There is no transfer of matter or energy involved. Alice's particle has not been physically moved to Bob; only its state has been transferred. The term "teleportation", coined by Bennett et al [51], reflects the indistinguishability of quantum mechanical particles. The teleportation scheme combines two "impossible" procedures. If we remove the shared entangled state from Alice and Bob, the scheme becomes classical teleportation, which is impossible as mentioned before. On the other hand, if the classical channel is removed, then it becomes an attempt to achieve superluminal communication, again impossible, via "no communication" theorem.

## 8.2 Future trends in Quantum Communication

The foregoing merely offers a 'taste' of the nature of quantum teleportation which is a rapidly growing field. Now, present-day quantum key distribution systems can operate only over the distances of several tens of kilometers, which may severely limit their practical applicability. due to various losses and noise in the communication lines (telecom optical fibers) and detectors. One of the major challenges in the field presently

is to develop long-distance quantum communication networks which allow secure quantum communication over arbitrarily long distances. Its main ingredients – the quantum memory for light, the source of entangled two-mode squeezed states, quantum teleportation, and entanglement swapping– have now been experimentally demonstrated in the laboratories. So a major task is the integration of these basic building blocks into a quantum network. This truly interdisciplinary effort requires a close collaboration of experimentalists with theoreticians and of quantum optics with atomic physicists. A very promising future direction is the so-called ”hybrid” quantum information processing which combines the approaches developed in the fields of single-photon linear optics QIP, and a corresponding QIP based on quantum continuous variables. The latter seem to be particularly suitable for quantum communication since they offer large bandwidth and may be easier to manipulate than quantum bits. And, unlike discrete atomic spins, distant atomic continuous-variable systems can be entangled and that a light-atoms quantum state exchange can be performed at the level of continuous variables. The resulting atomic quantum memory for light is thought to be a crucial component of the future quantum communication networks (see also ref [52]).

### 8.3 Emergence of Quantum Biology

For sometime scientists have been toying with ideas involving quantum phenomena [53] towards understanding the logic of ” Nature ” that always seems to have got around to do things in the most efficient and economical of ways. Also, thanks to nanotechnology, an increased understanding of mechanisms operating at the ” nano-level ” seem to have emboldened a rethink of existing paradigms : to consider the possibility of biological systems utilizing ” quantum weirdness ” ! In a highly suggestive paper, Patel [54] has drawn a parallel to Grover’s algorithm mentioned above [50] with that employed in DNA replication and protein synthesis. Of course quantum algorithmic mechanisms would require a sufficiently protected microenvironment for keeping decoherence effects at bay in the highly interactive cellular environment. Extensive experimental tests in concert with progress in our understanding of quantum information theory could someday bring out life’s secrets.

## 9 Retrospect And Conclusion

The foregoing is a rather sketchy description of the state of the art in the emerging field of foundations of quantum theory, The central issue is the ”conceptual anatomy” of quantum mechanics (bearing on quantum non-locality and the measurement process) that has sprung up to life, during the last 3 decades, in the context of fresh experimental prospects for its resolution.

Most of these efforts have been designed to throw light on the quantum measurement paradox, namely, *how* a definite outcome occurs in an individual measurement, (although standard quantum mechanics predicts a coherent superposition of different outcomes). A whole gamut of proposals since the Copenhagen Interpretation (see Sect 4) have come up to resolve the issue.

Another pathology is the so-called Zeno’s paradox [40] bearing on the effect of frequent measurements on the time of decay of an unstable system. Since it makes essential use



of the "collapse of state vector" , (one of the main contenders deemed responsible for the above dichotomy) an experimental resolution of Zeno's paradox [44] may well hold the key to a resolution of the measurement paradox. Finally the intriguing concept of quantum teleportation has opened up a most exciting field with vast possibilities for the future.

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